DISTRIBUTION OF STREAMS IN CHANNELS WITH POROUS WALLS

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The movement of a stream in a channel with permeable walls is examined on the basis of the energy equation. An analytical description is obtained for the distribution of the mean axial or radial velocity at the wall along the length of the channel.

In the design of many industrial devices the problem arises of the velocity distribution of the stream along the axis of a porous channel or of the calculation of the amount of material passing through its walls at any cross section. Three different approaches to the solution of this problem are known. The first of these, consisting in the use of the Bernoulli equation [1-3], has the drawback that it does not allow for energy effects arising during the addition or separation of mass to or from the stream. In a number of cases this results in large errors.

The second approach is based on the momentum theorem or the Meshcherskii equation of the dynamics of bodies of variable mass [4-8]. The one-dimensional problem is usually solved in this case, while an isolated element of the continuous viscous medium is essentially considered as a solid body. At the same time, the energy effects arising during the variation in the mass of the stream are manifested through the medium of viscous forces which perform the work in a continuous medium. The work of the viscous forces transforms the energy of the separating or combining mass into heat (dissipation) or mechanical energy (the restructuring of the axial velocity profile connected with the variation in flow rate). In the use of the Meshcherskii equation this mechanism of energy transformation escapes consideration, and in the calculating equations there appear coefficients whose use sometimes leads to physically absurd results [7]. If the movement of the stream is accompanied by thermal effects, such as in catalytic reactors or heat-exchange devices, the Meshcherskii equation becomes unsuitable in principle.

In [9-12] the movement of a stream in porous channels in a laminar mode is described by the Navier—Stokes equations. Solutions are obtained only for particular cases with the law of blowing or suction through the side walls of the channel assigned in advance. However, more often it is necessary to actually seek this law with the parameters of the surrounding medium in which the channel is placed being unknown.

Taking into account the observations made, it seems most natural to solve the formulated problem on the basis of the energy equation. The energy approach to a certain extent permits a more correct estimate of the effect of the momentum of the separating or combining mass on the base of the stream and, in addition, a more correct conversion to the one-dimensional problem and allowance for thermal effects when necessary.

The energy equation for a steady, isothermal, incompressible stream in a channel of arbitrary shape has the form [14]

$$\int_F \rho \mathbf{V}\left(\frac{V^2}{2}+\frac{P}{\rho}+gZ\right)d\mathbf{F} = \nu\rho\int_F V_j\left(\frac{\partial V_j}{\partial x_k}+\frac{\partial V_k}{\partial x_j}\right)dF_k - \nu\rho\int_\omega\left[\left(\frac{\partial V_j}{\partial x_k}\right)^2+\frac{\partial V_j}{\partial x_k}\cdot\frac{\partial V_k}{\partial x_j}\right]d\omega. \tag{1}$$

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The first integral on the right side of the equation expresses that part of the work performed by viscous forces which is expended in the formation of the profile in an unstabilized liquid stream.

The second integral on the right side of Eq. (1) expresses the dissipation of energy, while the integral on the left side expresses the flux of mechanical energy through the surface F bounding the elementary volume under consideration.

In the study of laminar streams with a variable flow rate the energy equation written for the instantaneous values of the velocity and pressure can be used directly in the form (1). For a turbulent stream it is necessary to resort to time averaging by the method of Reynolds [15].

Let us write the result of averaging the left side of Eq. (1):

$$\frac{1}{T}\int_0^T dt \int_F \rho \mathbf{V}\left(\frac{V^2}{2}+\frac{P}{\rho}+gZ\right)d\mathbf{F} = \int_F \rho\bar{\mathbf{V}}\left(\frac{V^2}{2}+\frac{P}{\rho}+gZ\right)d\mathbf{F} + \int_F [3/2\,\rho\bar{\mathbf{V}}(\bar{V}')^2 + \overline{\mathbf{V}'P'}]\,d\mathbf{F}. \tag{2}$$

Estimates show that the second integral on the right side of Eq. (2) is small in comparison with the first. The value $\overline{V'P'}/\rho$ is considerably less than the value $3/2\,(\overline{V'})^2$, which in turn does not exceed 6% of $\overline{V}^2/2$ on the average [15]. This fact makes it possible to neglect the integral involving the average pulsation values in Eq. (2) with an error of no more than 6%.

Performing the same operation on the right side of Eq. (1) and dropping the averaging notation, i.e., understanding V and P now to be the time-averaged values of these quantities, we obtain

$$\int_F \rho\mathbf{V}\left(\frac{V^2}{2}+\frac{P}{\rho}+gZ\right)d\mathbf{F} \approx \nu\rho\int_F V_j\left(\frac{\partial V_j}{\partial x_k}+\frac{\partial V_k}{\partial x_j}\right)dF_k - \nu\rho\int_\omega\left[\left(\frac{\partial V_j}{\partial x_k}\right)^2+\frac{\partial V_k}{\partial x_j}\cdot\frac{\partial V_j}{\partial x_k}\right]d\omega + \nu\rho\int_F V_j'\left(\frac{\partial V_j'}{\partial x_k}+\frac{\partial V_k'}{\partial x_j}\right)dF_k - \nu\rho\int_\omega\left[\left(\frac{\partial V_j'}{\partial x_k}\right)^2+\frac{\partial V_k'}{\partial x_j}\cdot\frac{\partial V_j'}{\partial x_k}\right]d\omega. \tag{3}$$

The last two integrals in Eq. (3) express the average work of restructuring of the pulsation velocity profile and the dissipation of pulsation energy, respectively.

The stated problem — finding the law of variation of the average velocity along the axis of a channel with perforated walls — is such that it is adequately solved in a one-dimensional approximation, averaging the quantities entering into Eq. (3) with respect to surface or volume.

If one considers a one-dimensional symmetrical stream with a variable flow rate in a horizontal cylindrical channel of radius r, then by integrating on the left side of Eq. (3) over the surface bounding the volume $\omega = \pi r^2 dx$ we obtain

$$\int_F \rho\mathbf{V}\left(\frac{V^2}{2}+\frac{P}{\rho}+gZ\right)d\mathbf{F} = \frac{\partial}{\partial x}\left[\int_0^{\pi r^2}\rho U\left(\frac{U^2}{2}+\frac{v^2}{2}+\frac{P}{\rho}\right)d\eta\right]dx + 2\pi r\varphi v_0\left(\frac{\varepsilon^2 w^2}{2}+\frac{v_0^2}{2}+\frac{p_0}{\rho}\right)dx = d\Phi. \tag{4}$$

Here $\varepsilon^2 w^2/2$ is the fraction of the kinetic energy of axial motion transported through the lateral surface of the channel and $d\Phi$ is the averaged right side of Eq. (3). With the averaging method adopted the value of Φ does not depend on the radial position at the end surface η.

Let us conduct the averaging operation over the surface η in Eq. (4), introducing the averaging corrections

$$\alpha_1 = \frac{\int_0^{\pi r^2} U^3 d\eta}{\pi r^2 w^3}, \quad \alpha_2 = \frac{\int_0^{\pi r^2} \rho U\left(\frac{v^2}{2} + \frac{P}{\rho}\right) d\eta}{\rho w \int_0^{\pi r^2} \left(\frac{v^2}{2} + \frac{P}{\rho}\right) d\eta}.$$

As a result of the integration and averaging over the surface of a differentially small volume on the left side of Eq. (3) we obtain

$$\int_F \rho \mathbf{V}\left(\frac{V^2}{2} + \frac{P}{\rho} + gZ\right) d\mathbf{F} = \pi r^2 \rho \frac{\partial}{\partial x}\left[\alpha_1 \frac{w^3}{2} + \alpha_2 w\left(\frac{v_0^2}{2} + \frac{p_0}{\rho}\right)\right] dx$$
$$+ 2\pi r \varphi \rho v_0 \left(\frac{\varepsilon^2 w^2}{2} + \frac{v_0^2}{2} + \frac{p_0}{\rho}\right) dx. \tag{5}$$

The first term on the right side of Eq. (5) is the longitudinal, and the second term the radial, energy flux through the lateral surface of the channel.

An analysis of numerous experimental data [4-8] shows that the averaging corrections α_1 and α_2 are very close to 1 and they can be taken as equal to 1 with an error acceptable for time averaging.

Furthermore, allowing for the observation concerning α_1 and α_2 and the condition of material balance $v_0 = -r/2\varphi \cdot \partial w/\partial x$, one can write (in a horizontal channel)

$$\int_F \rho \mathbf{V}\left(\frac{V^2}{2} + \frac{P}{\rho} + gZ\right) d\mathbf{F} = \pi r^2 \rho \left(\frac{3-\varepsilon^2}{2} w' w^2 + v_0' v_0 w + \frac{1}{\rho} p_0' w\right). \tag{6}$$

Here and below the prime, the first derivative of the corresponding value with respect to x is denoted.

Let us now turn to the right side of Eq. (3). We assume that the average dissipation of energy on the basis of well-known semiempirical theories can be expressed in the form

$$dN_{\text{dis}} = \tau_0 w 2\pi r dx = \rho \frac{\lambda w^3}{8} 2\pi r dx. \tag{7}$$

The coefficient of friction λ in a porous channel is a function of the radial velocity at the wall. Wallis [6] obtained the following dependence experimentally for the case of outflow:

$$\lambda = \lambda_0 + 3.56 \varphi \frac{v_0}{w}. \tag{8}$$

According to Wallis the energy dissipation is equal to

$$dN_{\text{dis}} = \left(\lambda_0 + 3,56\varphi \frac{v_0}{w}\right) \frac{\rho w^3}{8} 2\pi r dx. \tag{9}$$

It is seen from the latter equation that in comparison with movement in a channel with solid walls the additional energy losses caused by the presence of outflow have the proportionality coefficient $3.56\,\varphi(v_0/w)$. We further assume that in the average form the forces arising during the variation in flow rate in a porous channel are determined by the Meshcherskii equation [5]:

$$F = \pm (w - v_1) \frac{dm}{dt}. \tag{10}$$

The + sign denotes removal of mass and the — sign its addition to the flow. Here v_1 is the projection of the velocity of the removed or added mass onto the axis of motion while dm/dt is the amount of material being added or removed per unit time, equal in material balance to $\rho\,(\partial w/\partial x)\,\pi r^2 dx$. By introducing the notation $v_1/w = \varepsilon$ we can write the work of this force per unit time:

$$dA = \rho w^2 w' (1 - \varepsilon) \pi r^2 dx. \tag{11}$$

An important moment in the consideration of the motion of a viscous medium is the question of how this work is distributed between dissipation and reversible mechanical energy.

To a certain extent the answer to this question is contained in the work of Wallis mentioned. If it is assumed that all the work calculated from Eq. (11) is fully dissipated then by characterizing the energy

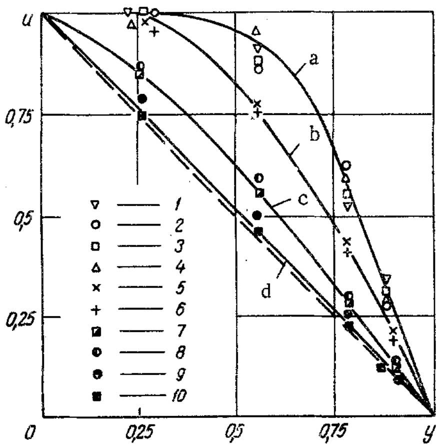

Fig. 1. Variation in average axial velocity of air along a perforated tube 0.5 m long and 0.106 m in diameter: a) $\varphi = 0.2$: 1) $Re_0 = 66{,}000$; 2) 78,100; 3) 83,400; 4) 120,300; b) $\varphi = 0.15$: 5) 77,300; 6) 120,300; c) $\varphi = 0.1$: 7) 77,600; 8) 99,400; d) $\varphi = 0.05$: 9) 79,800; 10) 120,300.

loss due to the outflow of mass through the coefficient λ_{0m} one can write (with $\varepsilon = 0$)

$$\lambda_{0m} = 8\varphi \frac{v_0}{w}. \tag{12}$$

Thus, the total work of the Meshcherskii forces expressed in the form of dissipation is equal to

$$dA = 8\varphi \frac{v_0}{w} \rho \frac{w^3}{8} 2\pi r dx. \tag{13}$$

According to the experiments of Wallis, based on Eq. (8) a part of this energy actually is dissipated $(3.56\,\varphi\,(v_0/w)\,\rho\,(w^3/8)\,2\pi r dx]$, and consequently the other part $(8-3.56)\,\varphi\,(v_0/w)\,\rho\,(w^3/8)\,2\pi r dx$ is mechanically reversible energy. Taking into account the observations made and Eq. (13), we can write the right side of Eq. (3) in the form

$$d\Phi = dA - dN_{\text{dis}} = 0.88\,\varphi \frac{v_0}{w} \rho \frac{w^3}{8} 2\pi r dx - \lambda_0 \rho \frac{w^3}{8} 2\pi r dx. \tag{14}$$

The quantity $0.88\,\varphi\,(v_0/w)\,\rho\,(w^3/8)\,2\pi r dx$ can be neglected, since the error introduced lies in the same range as the error allowed in averaging.

Now Eq. (3) can be written in the form

$$\frac{p_0'}{\rho} + \frac{3-\varepsilon^2}{2} w w' + v_0 v_0' + \lambda_0 \frac{w^2}{4r} = 0, \tag{15}$$

while in dimensionless form we will have

$$q' + \left(\frac{r}{2\varphi l}\right)^2 u'' u' + \frac{3-\varepsilon^2}{2} u' u + \frac{\lambda_0 l}{4r} u^2 = 0. \tag{16}$$

The equation obtained is distinguished by the fact that it does not contain a single empirical coefficient requiring determination by additional experiments.

The coefficient λ_0 is calculated from well-known equations as a function of the conditions. In order to finally obtain an equation describing the distribution of the average velocity along the channel it is necessary to supplement Eq. (16) with the appropriate outflow equation connecting the pressure inside and outside the channel.

A summary of the dimensionless second-order differential equations for the case of outflow into a quiescent medium such as the atmosphere is presented in our report [13]. There we also present some solutions, as well as the results of experiments with the laminar movement of water in a cylindrical channel 20 mm in diameter and 2 m long in the lateral surface of which there were capillaries intended for distribution of the stream.

The results of calculations and of experimental studies of certain types of radial catalytic reactors were published in [17].

In the present article we present only data concerning the movement of a turbulent stream in a perforated cylindrical channel with discharge into the atmosphere.

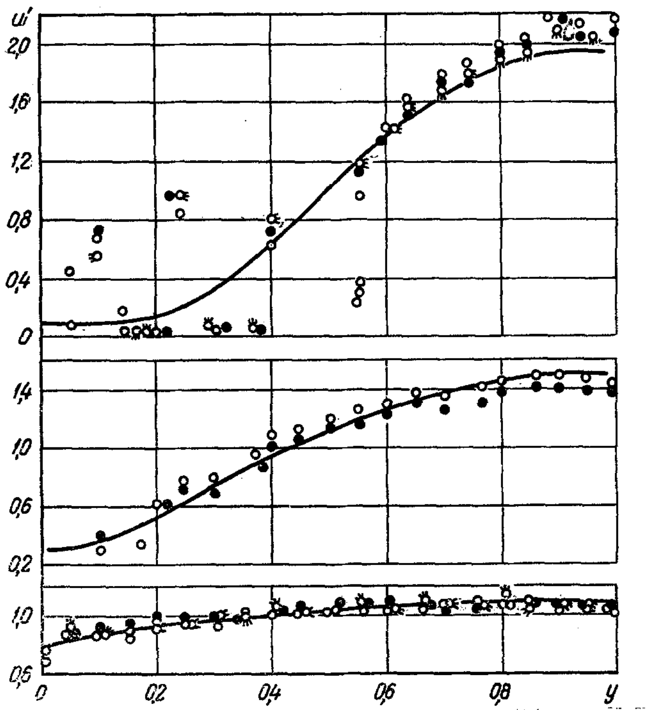

Fig. 2. Dimensionless flow rate through the lateral surface of a channel according to I. E. Idel'chik's data [18].

The differential equation describing the average velocity distribution along the channel axis has the form

$$u''u' + au'u + bu^2 = 0, \tag{17}$$

where

$$a = 2\,\frac{3-\varepsilon^2}{1+\xi}\left(\frac{\varphi l}{r}\right)^2,\quad b = \frac{\lambda_0\varphi^2}{1+\xi}\left(\frac{l}{r}\right)^3.$$

The solution for the boundary conditions

$$y = 0,\quad u = 1,$$
$$y = 1,\quad u = 0$$

(which corresponds to the end of the channel being blocked) is presented in [13].

As is seen, at large Reynolds numbers Re, when λ_0 = const, Eq. (17) is invariant with respect to Re and the coefficients a and b are constant.

The experimental apparatus consisted of a perforated tube 0.105 m in diameter with a preconnected section 20 diameters long to which up to 600 m^3/h of air was supplied. The presence of the preconnected section provided fully developed turbulent flow at the entrance to the perforated tube. The axial velocity profiles in several cross sections along the length of the tube were measured by means of Prandtl tubes shifted with a special coordinate device or by thermoanemometer pickups and the average velocity was calculated. Tubes with perforated sections 0.5 and 1.0 m long were used for the experiments. As an illustration, results are presented in Fig. 1 only for the tube 0.5 m long and 0.106 m in diameter. The measurement of the dimensionless velocity u averaged over the cross section is given along the dimensionless length y of the tube for different perforation of the lateral surface. The solid lines are constructed from Eq. (17) and the dashed line corresponds to the case of uniform distribution, when u' = —1. The variation in static pressure along the tube can be calculated from Eq. (16).

The experimental data are in satisfactory agreement with the theoretical curves for different initial Reynolds numbers Re_0, which confirms their invariance with respect to this parameter.

The qualitative and quantitative effect of variation in the open cross section of perforation and in the geometrical dimensions of the channel discovered experimentally is in agreement with the calculations. This is additionally confirmed by Fig. 2 in which the very detailed data of I. E. Idel'chik [18] are analyzed.

Idel'chik carried out his experiments using a channel of square cross section of 0.2×0.2 m and 6.3 m long. The theoretical curves in the figure (from top to bottom) correspond to increasing values of the coefficient of resistance ξ to the outflow of gas through the lateral surface of the channel, while the experimental points correspond to different values of Re_0.

We note that the dimensionless normal velocity at the wall, equal to the modulus of the derivative u', is laid out along the ordinate in Fig. 2. The errors always increase considerably when the derivative is used. Despite this, one cannot help noting the satisfactory agreement between I. E. Idel'chik's measurements and the theoretical curves.

The method presented in the present report for calculating the distribution of a stream in a porous channel has been tested in a rather wide range of the varying conditions. Good correspondence between the calculated functions and the experimental data was observed in all cases, which makes it possible to solve with sufficient reliability a number of problems of practical importance.

NOTATION

V: velocity vector of a moving elementary volume, m/sec; x: longitudinal coordinate, m; ρ: density of medium, kg/m^3; gZ: potential energy (energy of the gravitational field) per unit mass, m^2/sec^2; ω: elementary volume under consideration, m^3; F: surface bounding volume, m^2; ν: kinematic viscosity, m^2/sec; P: local pressure, N/m^2; p_0: pressure at channel wall, N/m^2; v: local radial velocity, m/sec; v_0: radial velocity at wall (of channel), m/sec; U: local axial velocity, m/sec; w: axial velocity averaged over a cross section, m/sec; φ: fraction of open cross section of lateral surface of channel; r: channel radius, m; l: channel length, m; q: dimensionless pressure; w_0: average velocity in initial cross section of channel, m/sec; u: dimensionless average axial velocity, $u = w/w_0$; y: dimensionless coordinate, $y = x/l$; q', u', u": corresponding derivatives with respect to dimensionless coordinate; λ_0: coefficient of friction during stream movement in a channel with solid walls; ξ: coefficient of resistance to outflow through lateral walls of channel.

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